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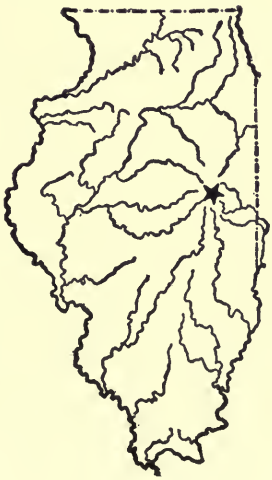
MEASURING THE BREEDING VALUE OF DAIRY SIRES BY THE RECORDS OF THEIR FIRST FEW ADVANCED REGISTRY DAUGHTERS

By F. A. Davidson



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MEASURING THE BREEDING VALUE OF DAIRY SIRES BY THE RECORDS OF THEIR FIRST FEW ADVANCED REGISTRY DAUGHTERS

By F. A. Davidson, First Assistant in Dairy Husbandry

INTRODUCTION

The breeding value of dairy sires lies mainly in their ability to transmit to their offspring factors for high milk and butterfat production. The fact that all sires are not equal in this respect makes it necessary to select for breeding purposes only those sires transmitting the largest number of such factors. An expression of this transmitting ability may be found in part in the milk and butterfat productions of the daughters, and it is by means of these productions that breeders in general have measured the breeding value of dairy sires. This method of measuring breeding values, altho used extensively at the present time, is open to several objections.

The productions of the daughters used in measuring the breeding value of dairy sires are those that are recorded in the advanced registers of the various purebred dairy cattle associations; daughters with records in these registers have fulfilled certain production requirements and are spoken of as tested daughters. The tested daughters of a sire, in most cases, represent only his best daughters, which are a small percentage of all his daughters. This selection is due in part to the requirements of the advanced registers, but in the main to the fact that under the present system of advanced registry testing it is exceedingly unprofitable to test daughters other than those showing signs of high producing ability. The productions of only the tested daughters, therefore, do not provide an absolute basis upon which to measure the breeding value of the sires. However, since the tested daughters of all sires have been subjected to the same type of selection, their productions provide a uniform basis and may be used as a means of measurement with the limitation that they will give only relative results.

The primary object in measuring the breeding value of dairy sires is to determine whether or not they are increasing the productions of their daughters over that of their daughters' dams. Inasmuch as there is convincing evidence that the inherited producing ability of the daughters is influenced as much by their dams as by their sires, the productions of the daughters' dams as well as of the daughters should be taken into account. Since, however, in many dairy herds the productions of very few of the dams have been recorded, the recorded

productions of the daughters provide the only available means of measuring the sire's breeding value. Altho the productions of the daughters cannot be used alone to measure the actual breeding value of the sire, they can be used to determine the approximate breeding value.

Breeders in general have found that the average of the milk and butterfat productions of a large number of tested daughters may be used as a relative measure of the transmitting ability of the sire. The testing of a large number of daughters, however, necessitates the expenditure of a great deal of time and money, and would be impractical in the majority of dairy herds at the present time. Furthermore, very few sires are kept in herds long enough for many of their daughters to be tested, being sold or killed long before their breeding value is determined. Many good sires have been lost in this way and many more will be lost, unless some means is provided whereby their breeding value can be determined when their first few daughters are tested. *What is needed is a method which will enable breeders to predict, within given limits, the average production of a large number of tested daughters of a sire from the average production of his first few tested daughters.* Such a method will save breeders a great deal of time and money and will also be an important factor in improving the transmitting ability of the dairy sire population in general.

In view of these facts, an investigation was planned in which the milk and butterfat productions of the tested daughters of a large number of dairy sires were studied. The purpose of the investigation was to derive a method by which the productions of the first few tested daughters of a sire might be used as a criterion to measure his relative breeding value.

SOURCE OF DATA

The Register of Merit of the American Jersey Cattle Club lists the names of Jersey sires having three or more tested daughters. Under each sire's name is listed the names of his tested daughters. Volumes 1911 to 1921 inclusive of the Register of Merit contain 133 sires having fifteen or more tested daughters with yearly or long-time records. The Jersey Register of Merit was chosen as the source of data because it contained the largest number of sires having fifteen or more tested daughters with yearly records.

OUTLINE OF INVESTIGATION

The investigation as planned involved three steps:

1. The determination of a definite large number of tested daughters of a sire, the average of whose productions could be used as a relative measure of the sire's breeding value.

2. The determination of the smallest number of first tested daughters of a sire, the average of whose productions will closely approximate the average production of this determined large number of tested daughters.

3. The derivation of a method by which the average production of the determined small number of tested daughters of a sire may be used to predict within given limits the average production of the determined large number of tested daughters.

Before proceeding with the three steps outlined above, it was necessary to correct the recorded production of each daughter for the influence of age of cow and percentage of fat in the milk. The correction for the influence of age was necessary because, on the average, the production of a cow increases up to the age of eight or nine years and then gradually decreases. This influence of age on production has been demonstrated very clearly by Gowen (1919), Pearl (1919), and Brody, Ragsdale, and Turner (1923). The correction for age was accomplished by reducing the production record (milk and fat) of each daughter to the production at the age of maximum production, which for Jerseys is eight years and one month. The factors used for these corrections are given in Table 6. Owing to the fact that the percentage of fat in the milk produced changes only very slightly with the age of the cow (Gowen 1919), the age-correction factors derived for the milk yield can also be used to correct the fat yield.

The productions of the daughters as recorded in the Register of Merit include the pounds of milk and the pounds of butterfat produced, both of which are needed to secure a complete expression of each daughter's production.[1] Therefore, it was necessary to find a single expression for both products. Such an expression may be secured by reducing the milk and butterfat productions of all the daughters to milk with a common percentage of fat. If all the milk productions of the daughters contain the same percentage of fat, the amount of fat in the milk will vary directly with the milk yield and need not be considered separately. By using the formula $.4\,M + 15\,F$, where $M =$ recorded milk yield in pounds and $F =$ recorded fat yield in pounds, all milk with varying fat percentages may be reduced to milk containing 4 percent of fat (indicated as F. C. M., fat-corrected milk).[2]

In the statistical study which follows, the recorded productions of all the daughters have been corrected, as explained above, both for influence of age of cow and for percentage of fat in the milk; that is, all productions are reported in terms of A.-F. C. M. (age- and fat-corrected milk).

[1]Milk contains solid material other than fat, collectively known as solids-not-fat. It requires work on the part of the cow to produce these solids; hence they should be taken into consideration as well as the fat when measuring her producing ability.

[2]This formula was determined by Gaines and Davidson (1923) and is based on the energy value of milk at varying fat percentages. Bulletin 245 of this Experiment Station gives a complete discussion of the derivation of this formula.

NUMBER OF TESTED DAUGHTERS NECESSARY TO MEASURE RELATIVE BREEDING VALUE OF SIRE

From a preliminary statistical study of the production records of the tested daughters of dairy sires it was found that the mathematical constants measuring the variability in the productions of the first fifteen tested daughters were in every case many times their probable errors and were changed only very slightly by the productions of additional daughters. The mathematical constants measuring the variability in the productions of the first fifteen tested daughters of ten of the 133 Jersey sires chosen at random are reported in Table A.

TABLE A.—MEASURES OF VARIABILITY IN THE A.-F.C.M. PRODUCTIONS OF THE FIRST FIFTEEN TESTED DAUGHTERS OF TEN JERSEY SIRES CHOSEN AT RANDOM FROM THE 133 JERSEY SIRES

Name of sire	Mean (pounds of A.-F.C.M.)	Standard deviation (pounds of A.-F.C.M.)	Coefficient of variability (percentage)
Sayda's Heir 3d	11483 ± 347	1990 ± 245	17.3 ± 2.2
Gamboge's Knight	11417 ± 271	1556 ± 192	13.6 ± 1.7
Hood Farm Torono	12183 ± 375	2152 ± 265	17.7 ± 2.2
Loretta's King	10250 ± 252	1449 ± 178	14.1 ± 1.8
Pogis 99th of Hood Farm	14383 ± 583	3349 ± 412	23.3 ± 3.0
Raleigh's Fairy Boy	10783 ± 245	1408 ± 173	13.1 ± 1.6
Royal Majesty of St. Cloud	10917 ± 374	2150 ± 265	19.7 ± 2.5
Hood Farm Pogis 9th	10450 ± 269	1547 ± 191	14.8 ± 1.9
Imp. Oxford You'll Do	10417 ± 358	2055 ± 253	19.7 ± 2.5
Irene's King Pogis	9983 ± 252	1448 ± 178	14.5 ± 1.8

These constants in Table A are also many times their probable errors. Hence it may be assumed that the variability in the productions of the first fifteen tested daughters of a Jersey sire is representative of the variability in the productions of any larger number of his tested daughters. Accordingly, the average production of the first fifteen tested daughters of a Jersey sire may be used as a relative measure of his breeding value and also as a basis with which may be compared the average production of any smaller number of his tested daughters.

SMALLEST NUMBER OF TESTED DAUGHTERS WHOSE AVERAGE PRODUCTION WILL APPROXIMATE AVERAGE OF LARGE NUMBER

In order to determine the smallest number of a sire's first tested daughters the average of whose productions would closely approximate the average production of his first fifteen tested daughters, it was necessary to classify the tested daughters of each of the 133 sires in the following manner:

1. Each daughter of each sire was first classified chronologically, i.e., as the first, second, third, etc., daughter of the sire to appear in the Register of Merit.

2. According to the first classification, the daughters of each sire were then separated into fifteen daughter-groups. These daughter-groups were produced by cumulation of the daughters from each preceding group; i.e., the first daughter appearing in the Register of Merit formed the first group, the first and second daughters appearing in the Register formed the second group, the first, second, and third daughters, the third group, etc., until all fifteen daughters were formed into the fifteenth group. A tabular illustration of this classification may be found in Table B.

3. A further classification was then made by combining all the daughter-groups of the 133 sires into fifteen aggregates. The first aggregate composed all the first daughter-groups, the second aggregate composed all the second daughter-groups, etc., the fifteenth aggregate composing all of the fifteenth daughter-groups. This classification is also illustrated in Table B.

The further analysis of the tested daughters of the 133 sires consisted of comparisons between the A.-F. C. M. productions of the first few daughters of each sire and the A.-F. C. M. productions of the first fifteen daughters of the sire. These comparisons were made as follows:

1. The average A.-F. C. M. production or mean A.-F. C. M. milk yield of each daughter-group was determined, i.e., the productions or milk yields of the daughters in each group were added and the sum divided by the number of daughters in the group.[1] The mean milk yields of the daughter-groups in each aggregate were correlated with the mean milk yields of their respective fifteenth daughter-groups in the fifteenth aggregate. In other words, the mean milk yields of all the first daughter-groups were correlated with the mean milk yields of their respective fifteenth daughter-groups, the mean milk yields of all the second daughter-groups were correlated with the mean milk yields of their respective fifteenth daughter-groups, etc., until every daughter-group was correlated with its respective fifteenth daughter-group. The means of the fifteenth daughter-groups were in every case used as the basis with which were correlated the means of the other daughter-groups. By making this comparison, a series of fifteen correlation coefficients was set up which indicated the relation existing between the mean milk yields of the daughter-groups in each aggregate and the mean milk yields of their respective daughter-groups in the fifteenth aggregate.

2. The standard deviation of the milk yields of the individual daughters in each aggregate (Table B) was compared with the standard deviation of the milk yields of the individual daughters in the fifteenth aggregate. A similar comparison was made between the co-

[1]Hereinafter for the sake of convenience the term "milk yield" will be used interchangeably with the term "production." It must always be remembered, however, that "milk yield" refers here to age- and fat-corrected milk and not to the recorded milk yield.

TABLE B.—DIAGRAM SHOWING CLASSIFICATION OF THE TESTED DAUGHTERS OF THE 133 JERSEY SIRES

Sire No.	Daughter-groups[1] formed by cumulation of daughters from each preceding group														
	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th
1	1	1–2	1–3	1–4	1–5	1–6	1–7	1–8	1–9	1–10	1–11	1–12	1–13	1–14	1–15
2	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
3	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
4	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
5	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
6	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
7	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
8, etc.	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
133	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Total number of daughter groups	133	133	133	133	133	133	133	133	133	133	133	133	133	133	133
Total number of individual daughters	133	266	399	532	665	798	931	1064	1197	1330	1463	1596	1729	1862	1995
Aggregates by summation of daughter groups	1st	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th

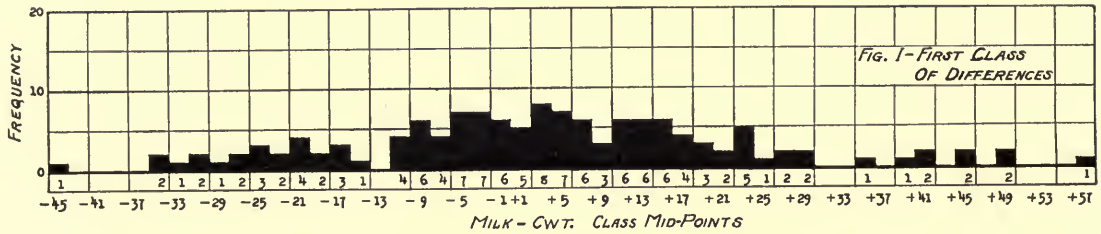
[1]The first group consisted of the first daughter to appear in the Register of Merit; the second group consisted of the first two daughters to appear in the Register of Merit; the third group consisted of the first three daughters to appear in the Register of Merit; etc.

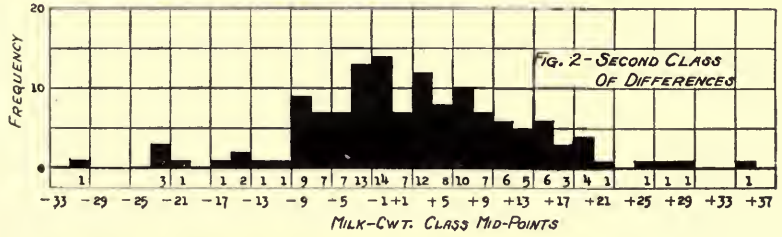
efficients of variability of the milk yields of the individual daughters in the aggregates. This comparison was necessary in order to determine whether or not the productions of the first few tested daughters of the 133 sires were more variable than the productions of the first fifteen tested daughters.

3. The mean milk yield of the fifteenth daughter-group of each sire was subtracted from the mean milk yield of each smaller group of tested daughters. The daughter-groups of the 133 sires were combined into aggregates; and the mean milk yields of the daughter-groups in the fifteenth aggregate were subtracted from the mean milk yields of their respective daughter-groups in each of the other aggregates. In this way a class of 133 differences was set up for each aggregate, there being 133 daughter-groups in each aggregate. The series of differences for each aggregate was given the same number as the aggregate from which it was determined, i.e., the differences between the mean milk yields of the daughter-groups in the fifteenth aggregate and the mean milk yields of their respective daughter-groups in the first aggregate formed the first class, etc. These classes of differences are shown graphically in Figs. 1 to 14. The mean and the standard deviation of each class of differences were determined. A comparison was then made between the means of the classes of differences and likewise between the standard deviations.

The mean of the first class of differences is equivalent to the difference between the mean milk yield of all the individual daughters in the fifteenth aggregate and the mean milk yield of all the individual daughters in the first aggregate. The same relation holds true for the mean of each class of differences. Hence the comparison between the means of the classes of differences indicates the relation existing between the mean milk yield of all the daughters in the fifteenth aggregate and the mean milk yield of all the daughters in each of the other aggregates.

The standard deviations of each class of differences measures the variability among the differences in each class. Hence a comparison between these standard deviations indicates the relation existing between classes in regard to the variability among the differences within them. In other words, this comparison indicates the extent to which the mean milk yields of the daughter-groups in each aggregate deviated from the mean milk yields of their respective daughter-groups in the fifteenth aggregate. It also provides another means of expressing the same relation as indicated by the correlations in the first comparison, i.e., it determines *on the average* how closely the mean milk yield of each daughter-group approximates the mean milk yield of the fifteenth daughter-group.


FREQUENCY
20
10
0
FIG. 1 - FIRST CLASS OF DIFFERENCES
1 2 1 2 1 2 3 2 4 2 3 1 4 6 4 7 7 6 5 8 7 6 3 6 6 6 4 3 2 5 1 2 2 1 1 2 2 2 1
-45 -41 -37 -33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33 +37 +41 +45 +49 +53 +57
MILK - CWT. CLASS MID-POINTS


FREQUENCY
20
10
0
FIG. 2 - SECOND CLASS OF DIFFERENCES
1 3 1 1 2 1 1 9 7 7 13 14 7 12 8 10 7 6 5 6 3 4 1 1 1 1 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33 +37
MILK-CWT. CLASS MID-POINTS

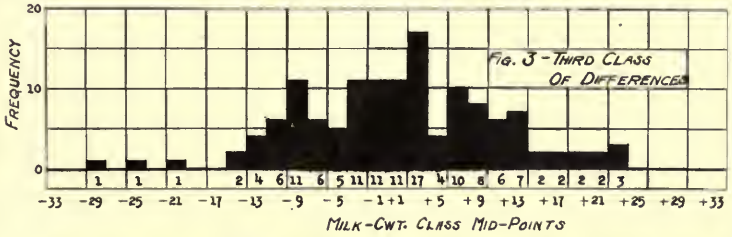


FIG. 3 - THIRD CLASS OF DIFFERENCES
FREQUENCY
20
10
0
1 1 1 2 4 6 11 6 5 11 11 11 17 4 10 8 6 7 2 2 2 2 3
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

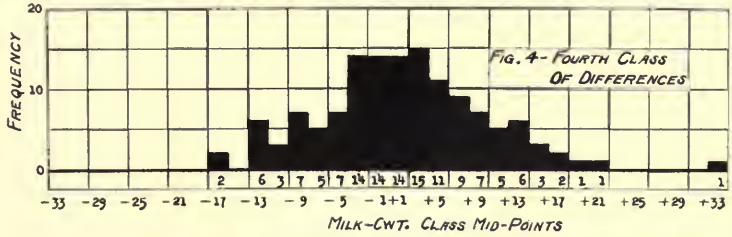


FIG. 4 - FOURTH CLASS OF DIFFERENCES
FREQUENCY
20
10
0
2 6 3 7 5 7 14 14 14 15 11 9 7 5 6 3 2 1 1 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

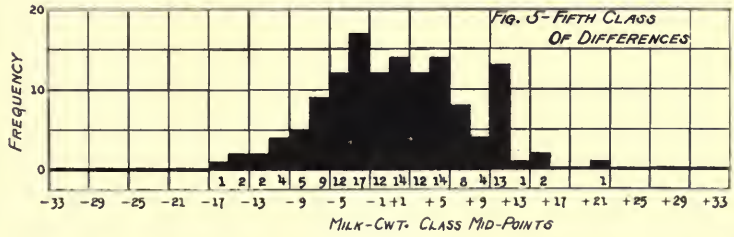


FIG. 5 - FIFTH CLASS OF DIFFERENCES
FREQUENCY
20
10
0
1 2 2 4 5 9 12 17 12 14 12 14 8 4 13 1 2 1 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

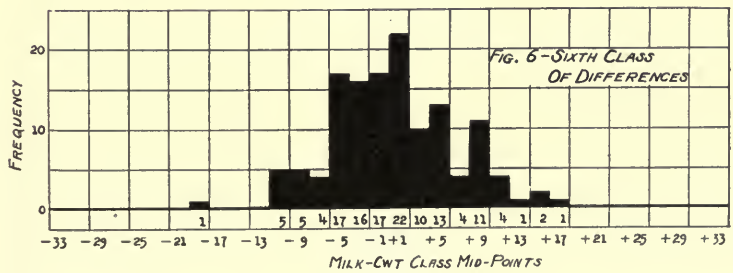


FIG. 6 - SIXTH CLASS OF DIFFERENCES
FREQUENCY
20
10
0
1 5 5 4 17 16 17 22 10 13 4 11 4 1 2 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT CLASS MID-POINTS

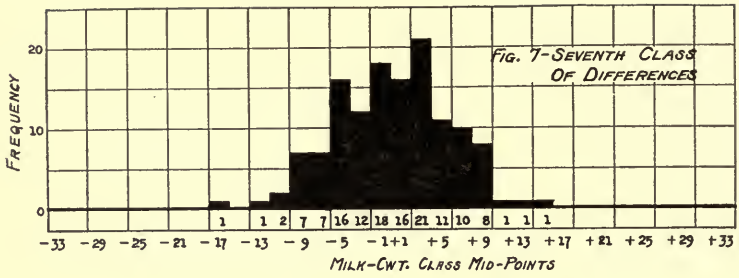


FIG. 7-SEVENTH CLASS OF DIFFERENCES
FREQUENCY
20
10
0
1 1 2 7 7 16 12 18 16 21 11 10 8 1 1 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

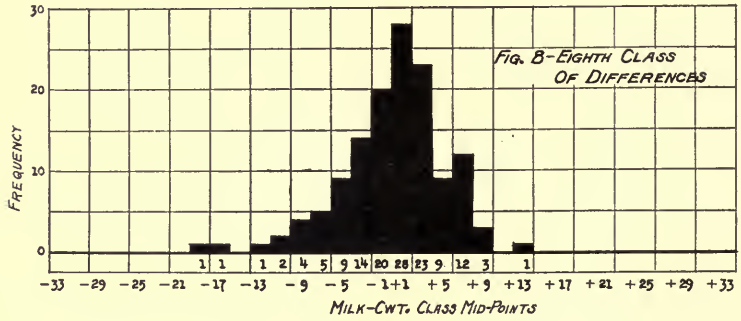


FIG. 8-EIGHTH CLASS OF DIFFERENCES
FREQUENCY
30
20
10
0
1 1 1 2 4 5 9 14 20 28 23 9 12 3 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

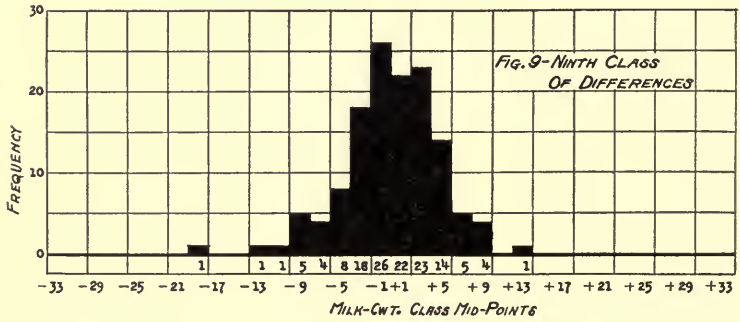


FIG. 9-NINTH CLASS OF DIFFERENCES
FREQUENCY
30
20
10
0
1 1 1 5 4 8 18 26 22 23 14 5 4 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

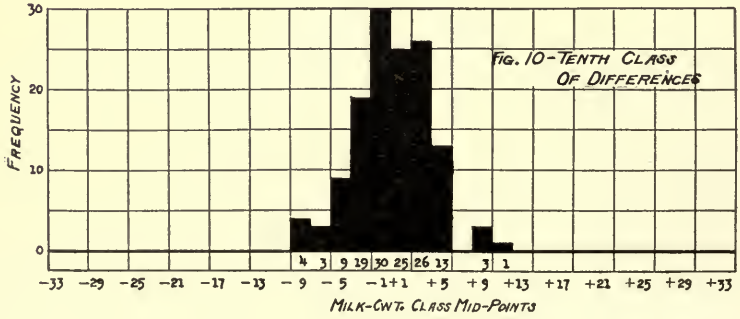


FIG. 10-TENTH CLASS OF DIFFERENCES
FREQUENCY
30
20
10
0
4 3 9 19 30 25 26 13 3 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

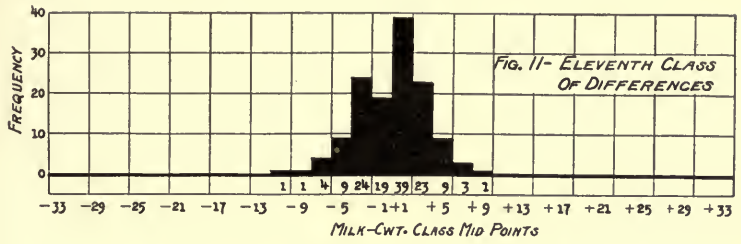


FIG. 11- ELEVENTH CLASS OF DIFFERENCES
FREQUENCY
0 10 20 30 40
1 1 4 9 24 19 39 23 9 3 1
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID POINTS

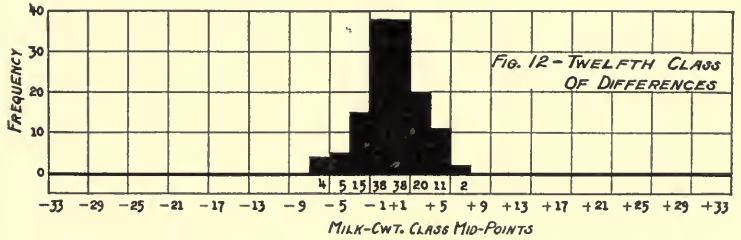


FIG. 12 - TWELFTH CLASS OF DIFFERENCES
FREQUENCY
0 10 20 30 40
4 5 15 38 38 20 11 2
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

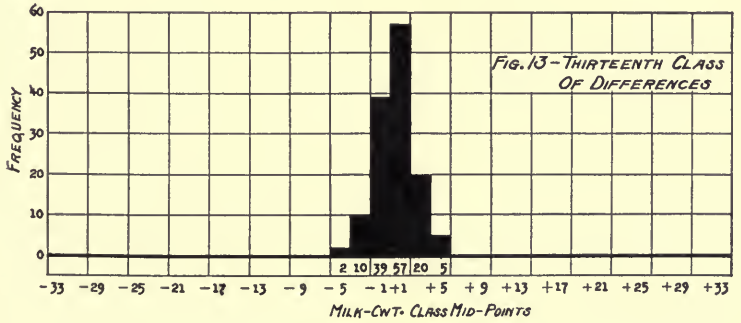


FIG. 13 - THIRTEENTH CLASS OF DIFFERENCES
FREQUENCY
0 10 20 30 40 50 60
2 10 39 57 20 5
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

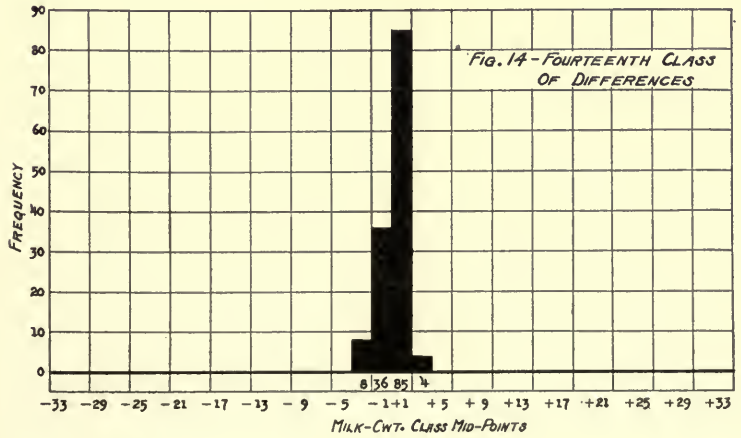


FIG. 14 - FOURTEENTH CLASS OF DIFFERENCES
FREQUENCY
0 10 20 30 40 50 60 70 80 90
8 36 85 4
-33 -29 -25 -21 -17 -13 -9 -5 -1+1 +5 +9 +13 +17 +21 +25 +29 +33
MILK-CWT. CLASS MID-POINTS

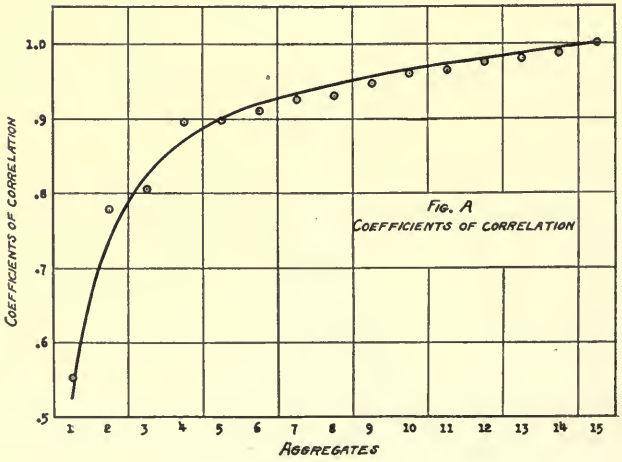
DISCUSSION OF RESULTS

The coefficients of the correlations between the mean milk yields of the daughter-groups in each aggregate and the mean milk yields of their respective daughter groups in the fifteenth aggregate, are reported in Table 1 and shown graphically in Fig. A. The fitted curve in Fig. A,

TABLE 1.—COEFFICIENTS OF THE CORRELATIONS BETWEEN THE MEAN MILK YIELDS OF THE DAUGHTER-GROUPS IN EACH AGGREGATE AND THE MEAN MILK YIELDS OF THE RESPECTIVE DAUGHTER-GROUPS IN THE FIFTEENTH AGGREGATE

Aggregates	Coefficients of Correlations	
	Observed	Calculated
1	.552 ± .041	.526
2	.777 ± .023	.737
3	.804 ± .027	.823
4	.895 ± .012	.869
5	.898 ± .011	.899
6	.910 ± .010	.919
7	.926 ± .008	.935
8	.930 ± .008	.947
9	.947 ± .006	.957
10	.959 ± .005	.965
11	.965 ± .004	.973
12	.976 ± .003	.981
13	.981 ± .002	.988
14	.987 ± .002	.994
15	1.000 ± .000	1.000

representing the relation between these coefficients, rises very rapidly at first and then gradually flattens out into an almost straight line. The point at which this flattening in the curve begins to be pronounced lies in the region of the 6 coefficient of correlation, which represents the



sixth daughter-groups. The equation to this curve is given in Table 7. In Table 1 the coefficients calculated from the fitted curve increase from +.526, the coefficient of the first aggregate, to +.919, the coefficient of the sixth aggregate, and from +.919, the coefficient of the sixth aggregate, to + 1.000, the coefficient of the fifteenth aggregate. Approximately three-quarters of the total increase in the correlations from the first to the fifteenth aggregate is reached by the sixth aggregate, thus showing that after the sixth aggregate additional daughters influenced the correlations only very slightly. The coefficient of + .919 for the sixth aggregate also represents a very significant correlation and, together with the above relation described by the fitted curve, indicates that on the average the mean milk yields of the first six tested daughters of the 133 sires are a very close approximation to the mean milk yields of their first fifteen tested daughters.

The standard deviations and coefficients of variability of the milk yields of the individual daughters in each aggregate are reported in Table 2 and shown graphically in Fig. B. The fitted curves in Fig. B, representing the relation between the standard deviations and the relation between the coefficients of variability, are straight lines which rise only very slightly from the first to the fifteenth aggregate. The equations to these curves are given in Table 7. The relations described by these fitted curves in Fig. B indicate that on the average the variability in the milk yields of the first six tested daughters of the 133 sires is practically the same as the variability in the milk yields of the first fifteen tested daughters. In fact, the variability in the milk yields of the tested daughters in any aggregate differs only very slightly from the variability in any other aggregate. Hence it may safely be assumed that the results from the first and third comparisons are not due to chance differences in the variability within the milk yields of the tested daughters in the aggregates.

The mean and the standard deviation of each class of differences are reported in Table 3 and shown graphically in Fig. C. The fitted curve in Fig. C, representing the relation between the standard deviations, falls very rapidly at first and then gradually flattens out into an almost straight line. Here again the point at which this flattening in the curve begins to be pronounced is in the region of the sixth standard deviation, which represent the sixth daughter-groups. The equation to this curve (Table 7) is of the same type as the equation to the curve expressing the relation between the coefficients of correlation in Fig. A. In Table 3 the means calculated from the fitted curve are very small and can hardly be said to indicate any general relation.

In the same table the standard deviations calculated from the fitted curve decrease from 1,829 pounds, the standard deviation of the first class of differences, to 602 pounds, the standard deviation of the sixth class of differences, and from 602 pounds, the standard deviation of the sixth class of differences, to 155 pounds, the standard deviation of the

TABLE 2.—MEASURES OF VARIABILITY IN THE MILK YIELDS OF THE INDIVIDUAL DAUGHTERS IN EACH AGGREGATE

Aggregates	Number of daughters	Mean (pounds of A.-F.C.M.)	Standard deviation (pounds of A.-F.C.M.)		Coefficient of variability (percentage)	
			Observed	Calculated	Observed	Calculated
1...	133	11754 ± 138	2277 ± 94.2	2314	19.4 ± .83	20.22
2...	266	11669 ± 94	2264 ± 66.2	2317	19.4 ± .59	20.23
3...	399	11578 ± 78	2312 ± 55.2	2320	20.0 ± .50	20.23
4...	532	11593 ± 70	2402 ± 49.7	2323	20.7 ± .45	20.24
5...	665	11515 ± 63	2392 ± 44.2	2325	20.8 ± .40	20.24
6...	798	11498 ± 57	2363 ± 39.9	2328	20.6 ± .36	20.25
7...	931	11478 ± 51	2324 ± 36.3	2331	20.3 ± .33	20.25
8...	1064	11472 ± 48	2319 ± 33.9	2334	20.2 ± .31	20.26
9...	1197	11463 ± 45	2312 ± 31.9	2336	20.2 ± .29	20.26
10...	1330	11469 ± 43	2319 ± 30.3	2339	20.2 ± .27	20.27
11...	1463	11464 ± 41	2315 ± 28.9	2342	20.2 ± .26	20.27
12...	1596	11482 ± 39	2323 ± 27.7	2345	20.2 ± .25	20.28
13...	1729	11499 ± 38	2364 ± 27.1	2347	20.6 ± .25	20.28
14...	1862	11475 ± 37	2355 ± 26.0	2350	20.5 ± .24	20.29
15...	1995	11453 ± 36	2362 ± 25.2	2353	20.6 ± .23	20.29

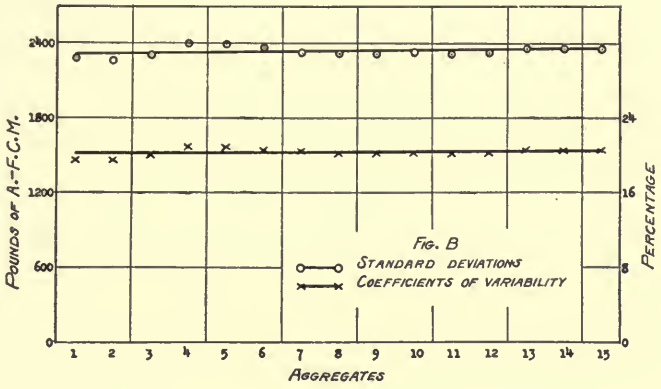
fourteenth class of differences. The relation between these standard deviations indicates that the variability in the differences between the mean milk yields of the daughter-groups in the fifteenth aggregate and the mean milk yields of their respective daughter-groups in each of the other aggregates decreases markedly from the first to the sixth aggregate and from there on decreases only very slightly. In other words, the first six tested daughters of the 133 sires is the smallest number of tested daughters whose mean milk yields do not deviate very widely from the mean milk yields of the first fifteen tested daughters. This

TABLE 3.—MEASURES OF VARIABILITY IN EACH CLASS OF DIFFERENCES[1]

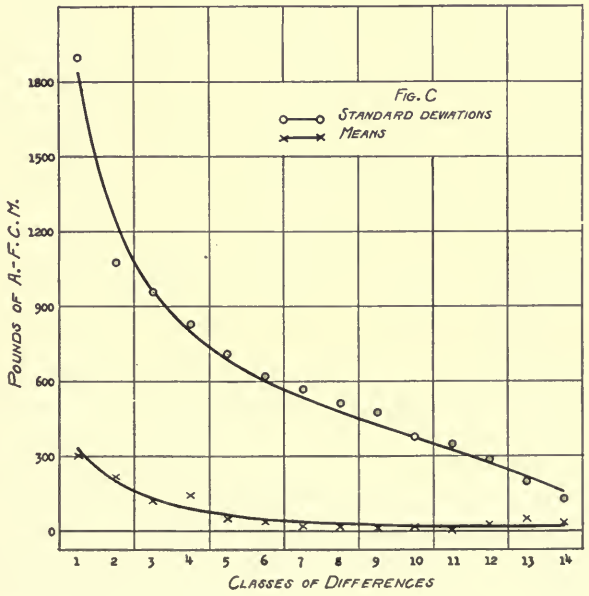
Class of differences	Mean (pounds of A.-F.C.M.)		Standard deviation (pounds of A.-F.C.M.)	
	Observed	Calculated	Observed	Calculated
1	+304	+321	1903 ± 78.7	1829
2	+219	+193	1070 ± 44.2	1252
3	+121	+129	970 ± 40.1	969
4	+144	+ 91	823 ± 34.0	801
5	+ 53	+ 66	707 ± 29.3	687
6	+ 38	+ 49	616 ± 25.5	602
7	+ 13	+ 36	566 ± 23.0	534
8	+ 13	+ 27	510 ± 21.1	476
9	+ 7	+ 21	471 ± 19.5	423
10	+ 13	+ 16	372 ± 15.4	372
11	+ 4	+ 14	342 ± 14.1	322
12	+ 23	+ 13	285 ± 11.8	269
13	+ 47	+ 13	196 ± 8.1	214
14	+ 28	+ 15	128 ± 5.3	155

[1]Each class of differences was determined by subtracting the mean milk yields of the daughter-groups in the fifteenth aggregate from the mean milk yields of the respective daughter-groups in each of the other aggregates.

relation is also illustrated very clearly by comparing the polygons in Figs. 1 to 14.



The mathematical constants in Tables 1, 2, and 3, which describe the relation existing between the milk yields of the first fifteen tested daughters of the 133 Jersey sires taken as a whole, are all many times



their probable errors. Hence it may be assumed that these tested daughters of the 133 Jersey sires, as classified into the daughter-groups, aggregates, etc., are representative of the corresponding tested daughters of all Jersey sires. Following this supposition it may also be assumed that *on the average* the above mathematical constants repre-

sent the relations existing between the milk yields of the first fifteen tested daughters of any Jersey sire. Accordingly then, a method can be derived by which the mean milk yield of the first fifteen tested daughters of any Jersey sire can be predicted to fall within given limits from the mean milk yield of any smaller number of his first tested daughters.

DERIVATION OF METHOD

Each class of differences, as shown graphically in Figs. 1 to 14, tends to approximate a normal frequency curve, some of the curves, however, being distinctly of the "cocked hat" type. The mean of any class of differences ± 2.14[1] times the standard deviation of that class gives limits such that the odds are 30 to 1 that any single difference,

TABLE 4.—LIMITS[1] AT ODDS OF 30 TO 1 AND 100 TO 1 FOR EACH GROUP OF FIRST TESTED DAUGHTERS OF ANY JERSEY SIRE

Daughter groups	Limits at odds of 30 to 1 (pounds of A.-F.C.M.)	Limits at odds of 100 to 1 (pounds of A.-F.C.M.)
1	+4235 to −3593	+5040 to −4398
2	+2872 to −2486	+3423 to −3037
3	+2203 to −1945	+2629 to −2371
4	+1805 to −1623	+2158 to −1976
5	+1536 to −1404	+1838 to −1706
6	+1337 to −1239	+1602 to −1504
7	+1179 to −1107	+1414 to −1342
8	+1046 to − 992	+1255 to −1201
9	+ 926 to − 884	+1112 to −1070
10	+ 812 to − 780	+ 976 to − 944
11	+ 703 to − 675	+ 845 to − 817
12	+ 589 to − 563	+ 707 to − 681
13	+ 471 to − 445	+ 565 to − 539
14	+ 347 to − 317	+ 415 to − 385

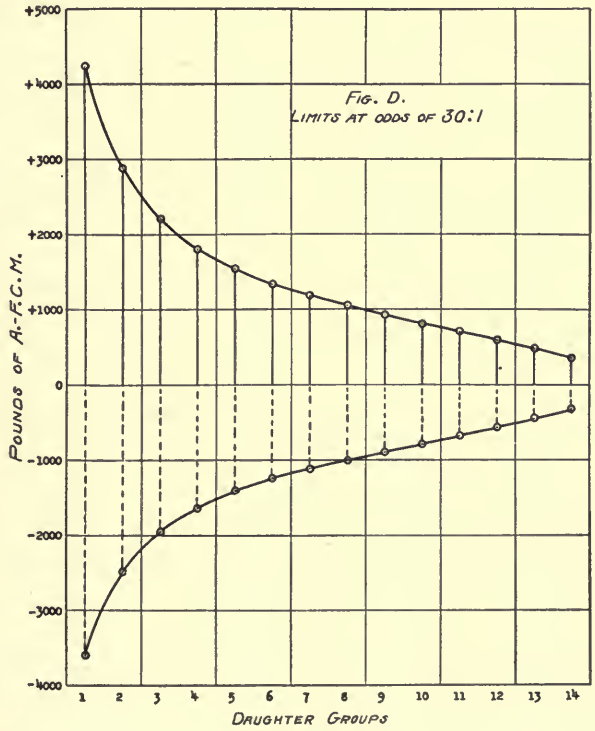
[1]These limits were determined from the calculated means and standard deviations of the classes of differences.

determined by the above methods for that class, will lie within them. Any single difference in a class of differences represents the difference between the mean milk yield of a corresponding daughter-group and the mean milk yield of the fifteenth daughter-group of *an individual sire*. Hence the mean of a class of differences ± 2.14 times the standard deviation of that class gives limits such that the chances are 30 to 1 that the difference between the mean milk yields of the corresponding group of tested daughters and the fifteenth group of tested daughters of *any Jersey sire,* will lie within them.[2] For example, the mean of the sixth

[1]The constant 2.14 was determined by means of the equation given on page XVIII of Karl Pearson's Tables for Statisticians.

[2]Odds of 30 to 1 are considered by most investigators as being great enough to be dependable; however, if greater odds are desired $2.58 \times \sigma$ will give odds of 100 to 1. The limits for these odds are also included in Table 5.

class of differences ± 2.14 times the standard deviation of that class, gives limits of + 1,337 pounds to — 1,239 pounds.[1] The chances are 30 to 1 then that the difference between the mean milk yield of the first fifteen tested daughters and the mean milk yield of the first six tested daughters of any Jersey sire will fall within them.[2] In other words, the chances are 30 to 1 that the mean milk yield of the first six tested daughters of any Jersey sire will not be more than 1,337 pounds above nor less than 1,239 pounds below the mean milk yield of his first fifteen tested daughters. In like manner similar limits were determined for each group of tested daughters. These limits are reported in Table 4 and shown graphically in Fig. D.



Hence limits may be found (Table 4) within which the chances are 30 to 1 that the mean milk yield of any small group of a sire's first tested daughters will deviate from the mean milk yield of his first fifteen tested daughters.

[1]The means and the standard deviations used to determine these limits were the calculated means and standard deviations given in Table 3.

[2]Any difference between the mean milk yield of the fifteenth group of tested daughters and the mean milk yield of any smaller group of tested daughters is always determined by subtracting the mean milk yield of the fifteenth group from the mean milk yield of the smaller group.

TABLE 5.—AGE-CORRECTION FACTORS FOR GUERNSEY MILK YIELDS

Age refers to the age of the cow at date of last calving preceding the commencement of the record.

(Additional months)	1 year	2 years	3 years	4 years
0	.....	1.43613	1.24875	1.14961
1	.....	1.41304	1.23840	1.14326
2	.....	1.39210	1.22854	1.13714
3	.....	1.37294	1.21910	1.13123
4	.....	1.35527	1.21007	1.12552
5	.....	1.33888	1.20142	1.12001
6	.....	1.32359	1.19312	1.11469
7	1.61443	1.30926	1.18514	1.10955
8	1.56493	1.29578	1.17748	1.10459
9	1.54841	1.28306	1.17011	1.09979
10	1.49109	1.27103	1.16302	1.09515
11	1.46191	1.25962	1.15619	1.09067
	5 years	6 years	7 years	8 years
0	1.08633	1.04453	1.01803	1.00374
1	1.08214	1.04179	1.01641	1.00304
2	1.07810	1.03916	1.01486	1.00242
3	1.07419	1.03663	1.01339	1.00186
4	1.07041	1.03420	1.01201	1.00139
5	1.06676	1.03186	1.01071	1.00098
6	1.06324	1.02961	1.00948	1.00064
7	1.05983	1.02746	1.00834	1.00038
8	1.05655	1.02538	1.00726	1.00018
9	1.05338	1.02342	1.00627	1.00005
10	1.05032	1.02154	1.00535	1.00000
11	1.04737	1.01974	1.00451	1.00001
	9 years	10 years	11 years	12 years
0	1.00010	1.00651	1.02312	1.05076
1	1.00025	1.00750	1.02499	1.05361
2	1.00047	1.00856	1.02693	1.05654
3	1.00076	1.00969	1.02894	1.05956
4	1.00113	1.01089	1.03105	1.06268
5	1.00155	1.01216	1.03322	1.06589
6	1.00205	1.01351	1.03548	1.06919
7	1.00263	1.01493	1.03782	1.07259
8	1.00326	1.01641	1.04024	1.07608
9	1.00397	1.01798	1.04274	1.07968
10	1.00475	1.01962	1.04533	1.08338
11	1.00559	1.02133	1.04800	1.08717
	13 years	14 years	15 years	16 years
0	1.09107	1.14678	1.22220	.....
1	1.09508	1.15224	1.22957	.....
2	1.09920	1.15784	1.23712	.....
3	1.10342	1.16358	1.24487	.....
4	1.10776	1.16946	1.25282	.....
5	1.11221	1.17549	1.26096	.....
6	1.11678	1.18167	1.26932	.....
7	1.12147	1.18802	1.27789	.....
8	1.12628	1.19452	1.28669	.....
9	1.13121	1.20118	1.29571	.....
10	1.13628	1.20802	1.30497	.....
11	1.14146	1.21502	1.31447	.....

EXPLANATION OF TABLE.—The factors given in this table were derived from the equation: $Y = 7017.4 + 328.0\ X - 12.4\ X^2 + 1729.2\ \text{Log}_{10}\ X$. ($Y$ = yield of milk in pounds; X = age in units of six months commencing at 1 year and 3 months for the zero point.) This equation was determined by the author, and is the equation of the curve expressing the influence of age on the milk yield of Guernsey cows. It is based upon 3,000 yearly milk records of cows as published in Vols. 12 to 34 of the Guernsey Advanced Register. According to the above equation, the milk yield at a given age multiplied by the factor given in the table for that age, reduces the yield to that of the age of maximum production, that is, to the yield at 8 years and 10 months.

TABLE 6.—AGE-CORRECTION FACTORS FOR JERSEY MILK YIELDS

Age refers to the age of the cow at date of last calving preceding the commencement of the records.

(Additional months)	1 year	2 years	3 years	4 years
0	2.19925	1.45349	1.25000	1.14012
1	2.03645	1.42959	1.23839	1.13327
2	1.92160	1.40766	1.22730	1.12663
3	1.83385	1.38735	1.21669	1.12020
4	1.76336	1.36836	1.20656	1.11408
5	1.70474	1.35044	1.19689	1.10816
6	1.65481	1.33351	1.18779	1.10241
7	1.61134	1.31752	1.17911	1.09709
8	1.57282	1.30259	1.17069	1.09182
9	1.53846	1.28833	1.16252	1.08684
10	1.50761	1.27486	1.15473	1.08202
11	1.47951	1.26215	1.14732	1.07735
	5 years	6 years	7 years	8 years
0	1.07285	1.03135	1.00817	1.00010
1	1.06849	1.02870	1.00695	1.00000
2	1.06440	1.02627	1.00583	1.00010
3	1.06045	1.02396	1.00482	1.00020
4	1.05664	1.02176	1.00392	1.00040
5	1.05296	1.01968	1.00311	1.00070
6	1.04943	1.01771	1.00241	1.00110
7	1.04603	1.01585	1.00170	1.00160
8	1.04286	1.01410	1.00120	1.00220
9	1.03972	1.01245	1.00080	1.00280
10	1.03681	1.01092	1.00050	1.00361
11	1.03402	1.00949	1.00020	1.00452
	9 years	10 years	11 years	12 years
0	1.00543	1.02428	1.05775	1.10853
1	1.00644	1.02648	1.06123	1.11359
2	1.00766	1.02881	1.06485	1.11894
3	1.00888	1.03125	1.06872	1.12435
4	1.01020	1.03370	1.07262	1.12994
5	1.01163	1.03627	1.07666	1.13572
6	1.01317	1.03907	1.08073	1.14168
7	1.01471	1.04188	1.08495	1.14784
8	1.01647	1.04482	1.08944	1.15407
9	1.01823	1.04789	1.09397	1.16050
10	1.02010	1.05108	1.09866	1.16727
11	1.02218	1.05441	1.10351	1.17412
	13 years	14 years	15 years	16 years
0	1.18120	1.28351	1.42939	.....
1	1.18835	1.29375	1.44418	.....
2	1.19574	1.30430	1.45946	.....
3	1.20351	1.31517	1.47521	.....
4	1.21153	1.32637	1.49151	.....
5	1.21966	1.33791	1.50840	.....
6	1.22790	1.34982	1.52583	.....
7	1.23655	1.36208	1.54386	.....
8	1.24533	1.37471	1.56254	.....
9	1.25455	1.38773	1.58187	.....
10	1.26390	1.40118	1.60191	.....
11	1.27355	1.41507	1.62267	.....

EXPLANATION OF TABLE.—The factors given in this table were derived from the equation: $Y = 4586.5 + 307.6\ X - 12.7\ X^2 + 2216.6\ \text{Log}_{10}X$. (Y = yield of milk in pounds; X = age in units of six months commencing at nine months for the zero point.) This equation was determined by John W. Gowen (Bul. 281, Maine Agr. Exp. Sta.) and is the equation of the curve expressing the influence of age on the milk yields of Jersey cows. It is based on 2,153 yearly milk records of cows as published in Vols. 1911 and 1913 of the Jersey Registry of Merit.

APPLICATION OF METHOD

In order to illustrate the accuracy of these limits and also their possible application to the tested daughters of Guernsey sires, five Jersey sires not included in the original 133 sires, and five Guernsey sires with fifteen or more tested daughters, were chosen at random, and the differences between the mean milk yields of their first fifteen tested daughters and the mean milk yields of each smaller group of first tested daughters determined by the above procedure. These differences are given in Table 8. The limits for each group of tested daughters, such that the chances are 30 to 1 that any of the above corresponding differences should fall within them, are also given in Table 8.

In comparing the difference for each group of daughters of the Jersey sires with its corresponding limits, it will be found that in every case the difference falls within them. These limits, therefore, appear to provide a dependable method by which the average productions (mean milk yields) of the first fifteen tested daughters of Jersey sires can be predicted from the average productions (mean milk yields) of any smaller number of their first tested daughters. From Table 8 it would seem that these limits may also be used in predicting the average productions of the first fifteen tested daughters of Guernsey sires from the average productions of any smaller number of their first tested daughters.

CAUTIONS IN USE OF METHOD

In using the limits set forth in Table 4, it must always be remembered that they refer to the corrected milk and butterfat productions of the tested daughters and not to their recorded productions. The method for correcting the recorded productions of the tested daughters of Guernsey and Jersey sires is described on page 547 and in Tables 5 and 6 of this bulletin. It must also be remembered that *on the average* the first six tested daughters of a sire is the smallest number of tested daughters the average of whose productions closely approximates the average production of his first fifteen tested daughters. The limits for smaller groups of tested daughters are much wider and only in exceptional cases would they be of much practical value.

CONCLUSIONS

The results from a statistical study of the variability within the corrected milk and butterfat productions of the tested daughters of 133 Jersey sires are as follows:[1]

1. In general the average production and variability among the productions of the first fifteen tested daughters of a Jersey sire are representative of the average production and variability among the productions of any larger number of the sire's tested daughters.

2. *On the average,* the first six tested daughters of a Jersey sire is the smallest number of tested daughters the average of whose produc-

[1]The recorded milk and butterfat productions of the tested daughters were corrected for age of cow and percentage of fat in the milk. See page 547 of text.

TABLE 7.—EQUATIONS TO FITTED CURVES IN FIGS. A, B, AND C

	Fig. A
Coefficients of correlations:	$Y = \frac{-.687}{X + .379} + 1.0256 + .000006X^3$
	Fig. B
Standard deviations:	$Y = 2311.4 + 2.765X$
Coefficients of variability:	$Y = 20.219 + .00511X$
	Fig. C
Standard deviations:	$Y = \frac{3129}{X + .882} + 164.7 - .08X^3$
Means:	$Y = \frac{769}{X + 1} - 63.6 + .01X^3$

NOTE.—X in every curve is a variable in units from 1 to 15, and Y is the variable whose quantity is determined by the equation.

tions closely approximates the average production of the first fifteen tested daughters of the sire.

3. A numerical measure of this closeness of agreement between the average productions of the first six tested daughters and the first fifteen tested daughters of a Jersey sire is as follows:

The chances are 30 to 1 that the average production of the first six tested daughters of a Jersey sire will not be more than 1,337 pounds above nor less than 1,239 pounds below the average production of the first fifteen tested daughters. In other words, this relation between the average productions of the first six tested daughters and the first fifteen tested daughters of a Jersey sire can be expected to hold true for 30 out of every 31 Jersey sires.

4. Breeders in general have found that the average production of a large number of tested daughters of a sire can be used as a relative measure of the sire's breeding value.[1] The first six tested daughters of a Jersey sire is the smallest number of first tested daughters whose average production can be used as a means to measure the approximate breeding value of the sire.

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[1]Owing to the equal influence of both the sire and dam upon the inherited producing ability of the daughters, and also to the fact that the tested daughters of a sire in most cases represent only the best daughters, this average can be used only as a relative measure of the breeding value of the sire. See page 545 of text.

TABLE 8.—DIFFERENCES BETWEEN THE MEAN MILK YIELD OF THE FIRST FIFTEEN TESTED DAUGHTERS AND THE MEAN MILK YIELD OF EACH SMALLER GROUP OF FIRST TESTED DAUGHTERS OF FIVE JERSEY[1] AND FIVE GUERNSEY SIRES CHOSEN AT RANDOM

Daughter groups	Limits within which the chances are 30 to 1 that any individual difference should fall	Jersey sires					Guernsey sires				
		Fontaine's Caiest	The Plymouth Lad	Prince of River Ridge	Financial King's Stockwell	Skamokawa Chief	Galaxy's Lavinius	Masher's Sequel	Governor of the Chene	Gypson's Count	Guiding Star
		Pounds of A.-F.C.M.									
1	+4235 to −3593	−1878	−2211	−1639	−1570	+2074	−2264	+1083	− 942	− 72	+1413
2	+2872 to −2486	−1045	−1852	− 958	− 177	+2159	−1850	− 164	− 147	+ 633	− 257
3	+2203 to −1945	− 463	− 870	− 535	+ 536	+1501	−2041	+ 535	− 584	− 97	−1001
4	+1805 to −1623	− 186	−1099	− 708	+ 606	+ 786	−1433	− 110	− 649	+ 444	− 894
5	+1536 to −1404	+ 93	− 746	− 439	− 93	+ 750	− 555	− 452	− 927	+ 362	− 925
6	+1337 to −1239	+ 91	− 233	− 366	− 691	+ 392	+ 50	− 213	+ 164	+ 323	− 896
7	+1179 to −1107	+ 120	+ 55	− 497	− 911	+ 154	− 21	− 98	+ 141	+ 237	− 985
8	+1046 to − 992	− 155	− 109	− 633	− 570	+ 184	+ 707	− 199	+ 66	+ 212	−1128
9	+ 926 to − 884	+ 210	− 272	− 565	− 287	+ 63	+ 710	− 422	− 43	+ 92	−1145
10	+ 812 to − 780	+ 178	− 188	− 429	− 139	+ 16	+ 551	+ 79	− 184	− 77	−1115
11	+ 703 to − 675	+ 164	− 370	− 327	− 277	+ 98	+ 258	− 26	− 372	+ 83	− 632
12	+ 589 to − 563	+ 49	− 473	− 354	− 173	+ 59	+ 187	− 201	− 178	+ 90	− 274
13	+ 471 to − 445	− 144	− 98	− 268	− 15	− 17	+ 123	− 243	− 138	− 39	+ 53
14	+ 347 to − 317	+ 111	+ 178	− 179	+ 11	− 38	+ 100	− 285	− 91	+ 49	+ 74

[1]Not included in original 133 Jersey sires.

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